Answers to Exercises 3.4.1:

Given the vectors $\mathbf{a} = (1, 2, 2)$, $\mathbf{b} = (1, 2, -2)$, and $\mathbf{c} = (2, 1, 2)$ in $\mathbb{R}^3$.

1. Calculate a) $\mathbf{a} \cdot \mathbf{b}$ and $\mathbf{a} \wedge \mathbf{b}$

   b) $\mathbf{ab} = 1 - 4\mathbf{e}_{13} - 8\mathbf{e}_{23}$.

   c) $(\mathbf{ab})\mathbf{c} = -6\mathbf{e}_1 - 15\mathbf{e}_2 + 18\mathbf{e}_3 - 12\mathbf{e}_{123}$

   d) $\mathbf{a}(\mathbf{bc}) = -6\mathbf{e}_1 - 15\mathbf{e}_2 + 18\mathbf{e}_3 - 12\mathbf{e}_{123}$

   e) $\mathbf{a}(\mathbf{b} + \mathbf{c}) = 9 - 3\mathbf{e}_{12} - 6\mathbf{e}_{13} - 6\mathbf{e}_{23}$

   f) $\mathbf{ab} + \mathbf{ac} = 9 - 3\mathbf{e}_{12} - 6\mathbf{e}_{13} - 6\mathbf{e}_{23}$.

2. a) Find the oriented area and magnitude of the bivector $\mathbf{a} \wedge \mathbf{b}$.

$$\frac{\mathbf{a} \wedge \mathbf{b}}{|\mathbf{a} \wedge \mathbf{b}|} = \frac{-4\mathbf{e}_{13} - 8\mathbf{e}_{23}}{4\sqrt{5}}, \quad |\mathbf{a} \wedge \mathbf{b}| = 4\sqrt{5}.$$

   b) Graph the bivector $\mathbf{a} \wedge \mathbf{b}$.

   c) Find the polar form of $\mathbf{ac}$

$$9\hat{\mathbf{a}}\hat{\mathbf{b}} = 9e^{\theta B}$$

where $\theta = \cos^{-1}\frac{8}{9}$, and $B = \frac{-3\mathbf{e}_{12} - 2\mathbf{e}_{13} + 2\mathbf{e}_{23}}{\sqrt{17}}$.

   d) $\sqrt{\mathbf{ab}} = 2.236 - 0.894\mathbf{e}_{13} - 1.789\mathbf{e}_{23}$ and
   $\sqrt{\mathbf{ac}} = 2.915 - 0.514\mathbf{e}_{12} - 0.343\mathbf{e}_{13} + 0.343\mathbf{e}_{23}$.

3. a) Find a reflection $L(\mathbf{x})$ and a rotation $R(\mathbf{x})$ which takes $\mathbf{a}$ into $\mathbf{b}$.

$$L(\mathbf{x}) = -(\mathbf{a} - \mathbf{b})\mathbf{x}(\mathbf{a} - \mathbf{b})^{-1}, \quad R(\mathbf{x}) = \sqrt{\mathbf{ba}^{-1}}\mathbf{x}\sqrt{\mathbf{a}^{-1}\mathbf{b}}.$$

Define the functions $L(\mathbf{x})$ and $R(\mathbf{x})$ in CLICAL and check that they have the desired properties.

   b) Find a reflection $L(\mathbf{x})$ and a rotation $R(\mathbf{x})$ which takes $\mathbf{b}$ into $\mathbf{c}$.

   Define the functions $L(\mathbf{x})$ and $R(\mathbf{x})$ in CLICAL and check that they have the desired properties.

   c) Find a reflection $L(\mathbf{x})$ and a rotation $R(\mathbf{x})$ which takes $\mathbf{a}$ into $\mathbf{c}$.

   Define the functions $L(\mathbf{x})$ and $R(\mathbf{x})$ in CLICAL and check that they have the desired properties.

4. Verify (3.17) for the rotation (3.15).

   Check with Clical for $\mathbf{x} = \mathbf{e}_i$ for $i = 1, 2, 3$.