MT 318 Differential Geometry **Universidad de Las Americas - Puebla** February 8, 2010

Quiz 4 (**20 pts.**), Name:——————————

Given the vectors $\mathbf{a} = \mathbf{e}_1$, and $\mathbf{b} = \frac{\mathbf{e}_1 + \sqrt{3}\mathbf{e}_2}{2}$.

1. **10 pts.** a) Find $\sqrt{\mathbf{ab}}$. $\sqrt{\mathbf{ab}} =$ ——————.

Answer: Since $\mathbf{a}$ and $\mathbf{b}$ are unit vectors,

$$\sqrt{\mathbf{ab}} = \mathbf{a}\frac{\mathbf{a}+\mathbf{b}}{|\mathbf{a}+\mathbf{b}|} = \mathbf{e}_1\frac{3\mathbf{e}_1 + \sqrt{3}\mathbf{e}_2}{\sqrt{12}} = \frac{3+\sqrt{3}\mathbf{e}_{12}}{\sqrt{12}} = \frac{3+\sqrt{3}\mathbf{e}_{12}}{2\sqrt{3}}.$$

b) Verify your answer in part a)

Answer:

$$\left[\frac{3+\sqrt{3}\mathbf{e}_{12}}{2\sqrt{3}}\right]^2 = \frac{9+6\sqrt{3}\mathbf{e}_{12}-3}{12} = \frac{1+\sqrt{3}\mathbf{e}_{12}}{2} = \mathbf{ab}.$$

2. **5 pts.** Find a rotation $R(\mathbf{x})$ such that $R(\mathbf{a}) = \mathbf{b}$. Give a figure for your result.

Answer:

$$R(\mathbf{x}) = \frac{1-\sqrt{3}\mathbf{e}_{12}}{2}\mathbf{x}\frac{1+\sqrt{3}\mathbf{e}_{12}}{2}$$

3. **5 pts.** Find a reflection $L(\mathbf{x})$ such that $L(\mathbf{a}) = \mathbf{b}$. Give a figure for your result.

Answer:

$$L(\mathbf{x}) = -(\mathbf{a}-\mathbf{b})\mathbf{x}(\mathbf{a}-\mathbf{b})^{-1} = -\frac{(\mathbf{e}-\sqrt{3}\mathbf{e}_2)\mathbf{x}(\mathbf{e}_1-\sqrt{3}\mathbf{e}_2)}{4}.$$